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THE CIRCLE SQUARED

A SOLUTION

OF THE

PROBLEM OF SQUARING THE CIRCLE

Showing the Exact Ratio between

RADIUS AND QUADRANT

BY

Ebenezer Erskine Wallace

E. E. WALLACE
MONMOUTH, ILLINOIS
1903

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PREFACE

Up to the present time the problem of finding the exact ratio between the diameter and the circumference of a circle has remained unsolved. For more than one hundred years the leading scientific societies of the world have declared the problem to be impossible of solution. It is, therefore, with some diffidence that I submit the following pages to the consideration of mathematicians and others interested in the solution of a problem heretofore deemed unsolvable. And yet, having worked on the problem for a period of time covering more than fifteen years, it is with considerable assurance of its correctness that I submit the solution, as I believe it to be.

I may be pardoned for such assurance when it is considered that I have worked out the problem by two separate and distinct theorems and find the answers to be the same to six decimal places. This would be impossible if either theorem were incorrect. The answer thus obtained for the ratio of the circumference to the diameter to six decimal places is 3.141941+. By calculating upon the old theory of approximation and subdividing the sides until I had over three million sides to a polygon inside of a circle, I obtained the answer 3.1418+, which tends to show that the theorems herewith submitted are correct.

I am well aware that the correctness of these theorems will be called in question, and if they are not mathematically correct they must fall. I submit them, however, with the confident belief that the time will come when they will be universally recognized as the proper solution of this problem.

EBENEZER ERSKINE WALLACE.

MONMOUTH, ILLINOIS, May 18, 1903.

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TABLE A

Sides			Remainders or Loss	
6 ×	0	.5	----------	= 3.
12 ×	1	.258819045102520	8460462834	= 3.105828541230240
24 ×	2	.130526183513181	3899039747	= 3.132628404316344
48 ×	3	.065403124924338	0515367387	= 3.139349996368224
96 ×	4	.032719080665409	2579935404	= 3.141031743879264
192 ×	5	.016361730547870	1772725885	= 3.141452265191040
384 ×	6	.008181132952953	7982770512	= 3.141555053934336
768 ×	7	.004090603756546	7977365189	= 3.141583685027328
1536 ×	8	.002045306156319	2273950508	= 3.141590256105984
3072 ×	9	.001022653612915	7796943436	= 3.141591898874880
6144 ×	10	.000511326873302	4221834471	= 3.141592309567488
12288 ×	11	.000255663445004	----------	= 3.141592412233728
24576 ×	12	.000127831723547	8350493851	= 3.141592437891072
49152 ×	13	.000063915861904	4732978108	= 3.141592444305408
98304 ×	14	.000031957930970	0447026860	= 3.141592446074880
196608 ×	15	.000015978965486	3191307784	= 3.141592446201488
393216 ×	16	.000007989482743	4208153432	= 3.141592446271488
786432 ×	17	.0000039947413717	----------	= 3.141592446428774
1572864 ×	18	.00000199737068586	----------	= 3.141592446453940224
3145728 ×	19	.000000998685342937	----------	= 3.141592446466523136

TABLE B

Side is to Side		As Loss is to Loss		
17	1	1	17	.000000000054815286108822 0
17	2	2	17	.000000000012739918070222 1
17	3	3	17	.000000000000843778487008 6
17	4	4	17	.000000000002113106125162 9
17	5	5	17	.000000000000276076122878 8
17	6	6	17	.000000000002044447089651 9
17	7	7	17	.000000000000816879917339 7
17	8	8	17	.000000000000116421178275 5
17	9	9	17	.000000000001996017172135 0
17	10	10	17	.000000000000054039478856 5
17	11	11	17	----------
17	12	12	17	.000000000000026721593274 6
17	13	13	17	.000000000000007572764968 5
17	14	14	17	.000000000000000357371163 6
17	15	15	17	.000000000000001276545644 6
17	16	16	17	.000000000000000805106655 7
--	--	--	--	.000000000007585270311206 00

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FIRST THEOREM

Analytic Study of the Relation of the Radius to the Circumference

There is a true ratio existing between the radius and the quadrant of the circle, and it can be found as truly as the hypotenuse of a right angle triangle when the two sides are known. Fig. 1, Theorem First, and the following example worked out to four decimal places will fully explain.

I submit this proposition:

The square of the sum of the two sides AC and BC is to the difference of their squares (which is square of radius) as the sum of the two sides AC and BC is to their difference. Then:

To find the quadrant BEC add BD square + DC square − EF square to the square of the sum of the two sides AC and BC; for want of a better term, call it the square of the curve.

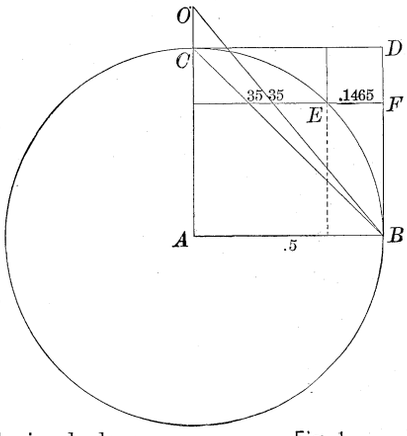

Fig. 1.

Solution of the proposition to four decimal places.

```
AC =   .50
BC =   .7071
       ------
       1.2071
       1.2071
       ------
        12071
       84497
      24142
    1.2071
    ----------
```

1.45709041 square of the sum of the two sides AC and BC.

Find EF by taking $\frac{1}{2}$ the square of radius AB $= .5 \times .5 = \frac{.25}{2} = .125$

```
√.125 (.3535
      9
     ----
65 ) 350
     325
     -----
703 ) 2500
      2109
      ------
7065 ) 39100
       35325
       ------
```


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Then BD square + DC square = .50
− .3535
.1465 = EF
.1465
7325
8790
5860
1465
.02146225

BD square + DC square = .50
− EF square = .02146225
.47853775

The square of the sum of the two other sides = 1.45709041
+ BD square + DC square − EF square = .47853775
1.93562816

$\sqrt{1.93562816}$ ⌊ 1.3912 sum of the two sides.
1
23 ⌡ 93
69
269 ⌡ 2456
2421
2781 ⌡ 3528
2781
74716

The square of the sum of the two sides AC and BC = 1.45709041
+ BD square + DC square − EF square = .47853775
1.93562816

Then sq. of sum of 2 sides : sq. of radius : : sum of 2 sides : diff. of 2 sides
1.93562816 : .25 : : 1.3912 : .1796

.25
69560
27824
.347800000 ⌊ .1796
.193562816
1542371840
1354939712
1874321280
1742065344
1322559360
1161376896

Sum of the two sides, ... 1.3912
Difference of the two sides, . . .1796
2 ⌡ 1.2116
.6058
.1796
Quadrant of circle, ...7854
4
3.1416

By calculating this proposition to fifteen decimal places the result will be 3.1419413724.

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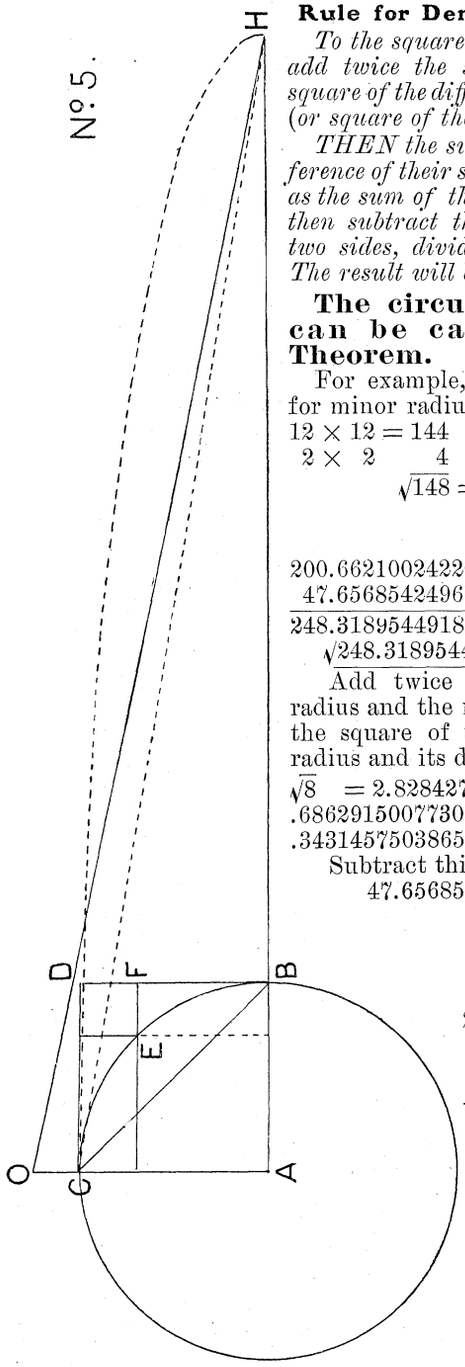
No. 5.



Rule for Demonstrating First Proposition

To the square of the sum of radius and diagonal add twice the square of radius minus half the square of the difference between radius and diagonal (or square of the curve).

THEN the sum of the squares will be to the difference of their squares (which is square of radius) as the sum of the two sides is to their difference; then subtract the difference from the sum of the two sides, divide by two, and add the difference. The result will be the quadrant of the circle.

The circumference of the ellipsis can be calculated by the same Theorem.

For example, take 12 for greater radius and 2 for minor radius.

$12 \times 12 = 144$
$2 \times 2 \quad 4$

$\sqrt{148} = 12.16552506059$ = the diagonal.

$\underline{2.}$
$14.16552506059^2 =$
$200.66210024220332317114 81$ = square of sum of
$47.6568542496134880374 2$ [radius and diagonal.
$248.31895449181681120856 81$

$\sqrt{248.31895449181681120856 81} = 15.7581393093$

Add twice the mean square of the greater radius and the minor radius $2 \times 12 \times 2 = 48$ — half the square of the difference between the minor radius and its diagonal. $2 \times 2 = 4 \times 2 = 8$.

$\sqrt{8} = 2.8284271246 - 2 = .8284271246^2 =$
$.6862915007730239 2516 \div 2 =$
$.3431457503865119 6258$ (half square of difference)

Subtract this from 48 and result will be
$47.6568542496134880374 2$ = sq. of the curve.

Sq. of sum of 2 sides + sq. of curve : sq. of their difference (which is sq. of greater radius) : : sum of 2 sides : difference of 2 sides

248.318954491857 : 144 : :
15.7581393093 : 9.1381345623

$$\frac{15.7581393093 \times 144}{248.318954491857} = 9.1381345623$$

Sum of the two sides =
15.7581393093
Difference " = 9.1381345623
2) 6.6200047470
3.3100023735
9.1381345623
12.4481369358
4
Answer. 49.7925477432

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To make this solution of the question better understood, take the same example where 12 is the greater radius and 2 is the minor radius, as in Fig. 5.

AH = 12
AC = 2
12 × 12 = 144
2 × 2 = 4
148

BC = Diagonal of the smaller radius.

EF = One-half of difference between diagonal and smaller radius.

HO = The quadrant of the circumference.

```
√148 ⌊ 12.165525
  1
22 ⌋ 48
     44
241 ⌋ 400
      241
2426 ⌋ 15900
       14556
24325 ⌋ 134400
         121625
243305 ⌋ 1277500
         1216525
2433102 ⌋  6097500
           4866204
2433104 ⌋ 123129600
```

12.165525 = diagonal.
2. = smaller radius.
14.165525 = sum of two sides.

```
14.165525
 70827625
28331050
70827625
70827625
84993150
14165525
56662100
14165525
200.662098525625
 47.656855
```

248.318953 = sum of squares of two sides.

```
√248.318953 ⌊ 15.758139 = sum of the sides.
  1
25 ⌋ 148
     125
307 ⌋ 2331
       2149
3145 ⌋ 18289
        15725
31508 ⌋ 256453
         252064
315161 ⌋ 438900
          315161
3151623 ⌋ 12373900
           9454869
31516269 ⌋ 291903100
```


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Find diagonal BC Fig. 5.

```
AB=2×2=4      √ 8 ⌊2.828427
AC=2×2=4        4
              ----
          8   48 ⌋ 400
                   384
                 -----
              562 ⌋ 1600
                    1124
                  ------
             5648 ⌋ 47600
                    45184
                   -------
            56564 ⌋ 241600
                    226256
                   --------
           565682 ⌋ 1534400
                    1131364
                   ---------
                     402036
```

```
Diagonal 2.828427
-Radius  2.
         --------
           .828427
           .828427
         --------
           5798989
          1656854
         3313708
        6627416
       1656854
      6674162
     -----------
  2 ⌋ .686291294329
      ------------
      .343145647164 = half the
      square of difference of
      diagonal and radius.
```

```
12×2=24×2=48.
      -   .343145
          ---------
          47.656855
```

```
Then 248.318953 : 144 :: 15.758139 : 9.138134
                                 144
                              -------
                              63032556
                             63032556
                            15758139
                           ----------
                           2269.172016
          248.318953 ⌋ 2269.172016 ⌊ 9.138134 = difference of two sides.
                       2234870577
                       -----------
                         343014390
                         248318953
                         ---------
                          946954370
                          744956859
                          ---------
                         1142022110
                          993275812
                         ---------
                          148746298
```

```
15.758139=sum of two sides.
 9.138134=difference  "
 --------
2 ⌋ 6.620005
    --------
     3.310002
     9.138134
    --------
    12.448136=quadrant
            4     and
    --------
    49.792544=circumference
```

of the ellipsis whose greater radius is 12 and whose minor radius is 2.

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After calculating the area of the circle and its circumference according to First Theorem (Fig. 1), and also calculating the circumference of the ellipsis according to the same Theorem (Fig. 5), getting a little greater answer than is given in geometry, I made a careful calculation by approxi-

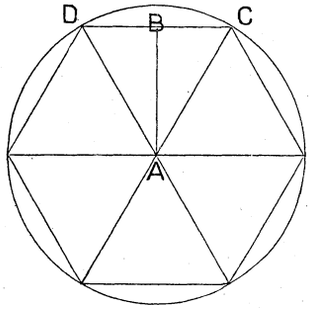

Fig. 7.

mation, taking a six-sided polygon inside of a circle whose diameter is 1 (Fig. 7), being divided into six segments, each and every side being equal to radius. Then DC $=.5\times6=3$, as per table of answers of subdivisions of sides calculated up to the sixteenth division .000007989482743$\times$393216 (number of sides)$=$3.141592446271488.

The remainder vanished after the sixteenth subdivision.

As all areas having like sides are in proportion to the squares of such like sides, so would all losses or remainders be in proportion to the squares of such sides. Look on table of answers up to the nineteenth subdivision calculated to fifteen decimal places. (Table A.) To the right I have carried out ten places more, calling them remainders or losses.

Table B will show calculations of the same.

By taking the seventeenth subdivision for the first term, calculating all the losses from Table A in the remainder column, adding them up I find .000000000075852703112060 0, as this is the loss in calculating one side, there being 786432$\times$.000000000075852703112060 0$=$

.000058653993029452209920 0	Losses added.
3.141592446271488	Answer at sixteenth subdivision.
3.141651100264517452209920	Answer.

If I had taken the eighteenth subdivision for the first term, the sides would have been doubled and the sides would have been diminished almost one-half, which would have increased the losses almost four times, so I will multiply

.000058653993029452209920	by 4
4	
.000234615972117808839 68	
3.141592446271488	
3.141827062243605	

This answer is obtained on the theory of obtaining the result by approximation, and I give it as evidence of the correctness of Theorem First, Fig. 1.

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SECOND THEOREM

I have found another solution of the same question, which I will endeavor to explain by calling attention to Fig. 6.

Taking a circle whose diameter is 1, construct a square AB touching the circle, each side of the square being equal to the diameter. Then construct a minor square within the circle marked GF Fig. 6.

Then $\sqrt{AB \times GF}$ = the quadrant of the circle. Multiply the quadrant of the circle CCCC by AB and the result obtained will be the area of the circle.

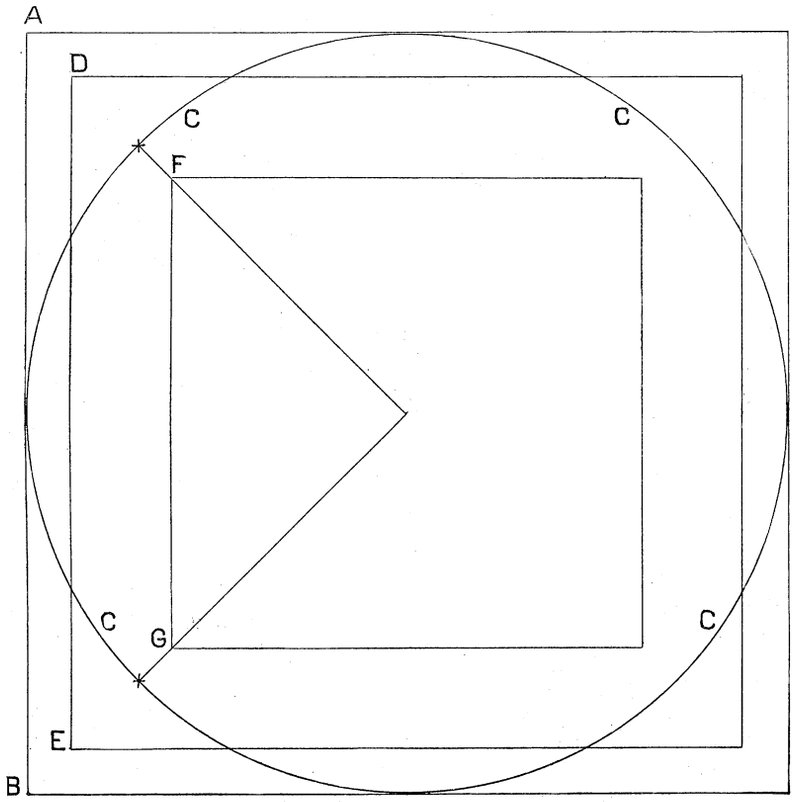

Fig. 6.

I find by inspection that six-sixteenths of 1 square (or outside square) =.375 square, which is a little too small by a little over .005. This difference is found by calculating the difference of the squares.

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The question arises why the factors $\frac{3}{8}$ and $\frac{5}{8}$ are used to obtain the foregoing difference. A circle whose diameter is 1 will have a radius $=.5$, and the square of radius $=.25$. Suppose the same diameter 1 is taken,
Let $.375 =$ smaller radius and
" $.625 =$ greater radius.
Then $.375 \times .625 = .234375 =$ square of mean radius.
Then $.25 - .234375 = .015625 = \frac{1}{4}$ BC square.
(Fig. 7) $\frac{.625}{4}$

Then the square of the radius of a perfect circle being .25, is to the square of the mean radius of the imperfect circle as the difference between the squares of the radius of the perfect circle and the imperfect circle (.234375) would be to the imperfect circle's difference, or ratio.
Then .25 : .234375 : : .015625 : .0146484375

```
      .015625
      -------
      1171875
      468750
     1406250
    1171875
    234375
  -----------
  .003662109375
```

```
.25 ) .003662109375 ( .0146484375
      25
      --
      116
      100
      ---
       162
       150
       ---
        121
        100
        ---
         210
         200
         ---
          109
          100
          ---
            93
            75
            --
            187
            175
            ---
             125
```

Then the mean between these two differences or ratios

$$.015625 \times .0146484375 = .000228881835937 5$$

Then $\sqrt{.000228881835937 5} = .015128841196122 72207$

The mean square of the two ratios or difference

$.015128841196122 72207 \div 16 = .000945552574757670$

```
                add  .0625
                     ----------------
                     .06344555257475767
                                     6
                     ----------------
                     .38067331544854602   Area of inside square
                                          as in former example.
```

I calculated the six-sided polygon inside of a circle whose diameter is 1 to over three million sides, and by bringing up the losses I find a result of 3.1418 +. But the two foregoing Theorems or Propositions are not by Approximation and are either true or false. I believe them to be true and correct.

EBENEZER ERSKINE WALLACE.

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